Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Tuesday 31 January 2006 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16) Mathematical Formulae (Lilac or Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other names and signature.
Check that you have the correct question paper.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 4 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

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1. A bead is threaded on a straight wire. The vector equation of the wire is

$$\mathbf{r} = \mathbf{i} - 3\mathbf{j} + \mathbf{k} + t(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}),$$

where the unit of length is the metre. The bead is moved from a point A on the wire through a distance of 6 m along the wire to a point B by a force $\mathbf{F} = (7\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$N.

Find the magnitude of the work done by $\mathbf{F}$ in moving the bead from A to B.

(Total 4 marks)

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2. A uniform circular disc has radius a and mass m. Prove, using integration, that the moment of inertia of the disc about an axis through its centre and perpendicular to the plane of the disc is $\frac{1}{2}ma^2$.

(Total 5 marks)

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3. The position vector $\mathbf{r}$ of a particle P at time t satisfies the vector differential equation

$$\frac{d\mathbf{r}}{dt} + 2\mathbf{r} = 4\mathbf{i}.$$

Given that the position vector of P at time $t = 0$ is $2\mathbf{j}$, find the position vector of P at time t.

(Total 6 marks)

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4. A uniform rod AB, of mass m and length $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis through A and perpendicular to the plane. The rod hangs in equilibrium with B below A. The rod is rotated through a small angle and released from rest at time $t = 0$.

(a) Show that the motion of the rod is approximately simple harmonic.

(4)

(b) Using this approximation, find the time t when the rod is first vertical after being released.

(2)

(Total 6 marks)

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5. A uniform circular disc has mass m and radius a. The disc can rotate freely about an axis that is in the same plane as the disc and tangential to the disc at a point A on its circumference. The disc hangs at rest in equilibrium with its centre O vertically below A. A particle P of mass m is moving horizontally and perpendicular to the disc with speed $\sqrt{(kga)}$, where k is a constant. The particle then strikes the disc at O and adheres to it at O. Given that the disc rotates through an angle of 90° before first coming to instantaneous rest, find the value of k.

(Total 10 marks)

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6. The vertices of a tetrahedron $PQRS$ have position vectors $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$ and $\mathbf{s}$ respectively, where

$$\mathbf{p} = -3\mathbf{i} + 4\mathbf{j} - \mathbf{k}, \qquad \mathbf{q} = 4\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}, \qquad \mathbf{r} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}, \qquad \mathbf{s} = 4\mathbf{i} + \mathbf{k}.$$

Forces of magnitude 20 N and $2\sqrt{13}$ N act along SQ and SR respectively. A third force acts at P.

Given that the system of three forces reduces to a couple $\mathbf{G}$, find

(a) the third force,

(6)

(b) the magnitude of $\mathbf{G}$.

(6)

(Total 12 marks)

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7. At time $t = 0$, a small body is projected vertically upwards. While ascending it picks up small drops of moisture from the atmosphere. The drops of moisture are at rest before they are picked up. At time t, the combined body P has mass m and speed v.

(a) Show that, while P is moving upwards, $m\dfrac{\mathrm{d}v}{\mathrm{d}t} + v\dfrac{\mathrm{d}m}{\mathrm{d}t} = -mg$.

(5)

The initial mass of P is M, and $m = Me^{kt}$, where k is a positive constant.

(b) Show that, while P is moving upwards, $\dfrac{\mathrm{d}}{\mathrm{d}t}(ve^{kt}) = -ge^{kt}$.

(3)

Given that the initial projection speed of P is $\dfrac{g}{2k}$,

(c) find, in terms of M, the mass of P when it reaches its highest point.

(7)

(Total 15 marks)

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8. Four uniform rods, each of mass m and length $2a$, are joined together at their ends to form a rigid square framework $ABCD$ of side $2a$. The framework is free to rotate in a vertical plane about a fixed smooth horizontal axis through A. The axis is perpendicular to the plane of the framework.

(a) Show that the moment of inertia of the framework about the axis is $\frac{40ma^2}{3}$.

(5)

The framework is slightly disturbed from rest when C is vertically above A. Find

(b) the angular acceleration of the framework when AC is horizontal,

(3)

(c) the angular speed of the framework when AC is horizontal,

(3)

(d) the magnitude of the force acting on the framework at A when AC is horizontal.

(6)

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

END